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PROPOSED GYROSCOPE EXPERIMENT TO TEST GENERAL RELATIVITY THEORY[*]

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There is a striking difference between the experimental
bases of the special and general theories of relativity. Special
relativity has been amply verified in several aspects: for example,
the dynamics of electrons and protons moving with speeds close to
that of light, the time dilation of the decay of rapidly-moving
π mesons, the classical radiation from fast electrons in magnetic
fields, and the predictions of relativistic quantum electrodynamics
with respect to bremsstrahlung and more subtle radiative processes.
In each of these categories, so many experiments have been found to
yield results in agreement with theoretical expectation (and none
in disagreement) that there can be no reasonable doubt as to the
correctness of special relativity as a description of natural
phenomena within its domain of validity. The situation is completely
different with general relativity. Here, there are thus far only
the three so-called "crucial tests": the gravitational red shift,

the deflection of starlight passing close to the sun, and the
precession of the perihelia of the orbits of the inner planets,
especially Mercury. And of these the first, which was recently
established in terrestrial experiments,[1] was shown by Einstein[2]
to follow directly from the equivalence principle, already
established experimentally by Eötvös,[3] without employing the
formalism of general relativity.

It is not surprising that it is so difficult to establish
the experimental superiority of Einstein's theory of gravitation
over that of Newton. Experimental situations that involve special
relativity require particles moving with speeds close to that of
light, and several kinds of such particles are plentifully produced
by modern accelerators. The corresponding situation in general
relativity would call for strong gravitational fields; the signifi-
cant parameter is GM/c^2r, where M is the mass of the gravitating
object, r the distance from its center, G the Newtonian constant
of gravitation, and c the speed of light. This parameter is
roughly 10^{-6} at the surface of the sun and 10^{-9} at the surface
of the earth. Thus available gravitational fields are very weak,
and Newtonian theory provides an excellent approximation.

It is because of this paucity of experimental information
that a new experiment was recently proposed.[4] This would consist
in moving a torque-free spherical gyroscope through the gravi-
tational field of the earth, and observing the precession of its

spin axis. According to Newtonian theory, there is no precession
of gravitational origin. However, the Einstein theory predicts
that the angular momentum vector $\underset{\sim}{S}_0$ of the gyroscope, measured
by a co-moving observer, would change with time in accordance
with the equations:

$$d\underset{\sim}{S}_0/dt = \underset{\sim}{\Omega} \times \underset{\sim}{S}_0 , \tag{1}$$

$$\underset{\sim}{\Omega} = (1/2mc^2)(\underset{\sim}{F} \times \underset{\sim}{v}) + (3GM/2c^2r^3)(\underset{\sim}{r} \times \underset{\sim}{v}) + (GI/c^2r^3)[(3\underset{\sim}{r}/r^2)(\underset{\sim}{\omega} \cdot \underset{\sim}{r}) - \underset{\sim}{\omega}]. \tag{2}$$

Here, m is the mass of the gyroscope, $\underset{\sim}{r}$ is its position vector
with respect to the center of the earth, $\underset{\sim}{v} = d\underset{\sim}{r}/dt$ is its velocity
vector, $\underset{\sim}{F}$ is any nongravitational force that may be applied to the
center of mass of the gyroscope, and M, I, and $\underset{\sim}{\omega}$ are the mass,
moment of inertia, and rotational angular velocity vector of the
earth.

Equations (1) and (2) were calculated[4] by means of the
dynamical method of Fock[5] and Papapetrou.[6] The first equation
shows that the magnitude of the spin angular momentum of the
gyroscope, and hence the rate of rotation measured by a co-moving
observer, is constant. It also shows that the direction of the
spin axis rotates with the vector angular velocity $\underset{\sim}{\Omega}$. The second
equation states that there are three parts to $\underset{\sim}{\Omega}$. The first term

does not involve M, and hence is not a gravitational effect; it
is the Thomas precession,[7] first discovered in the special rela-
tivistic treatment of atomic systems. The second term is the
geodetic precession caused by motion through the gravitational
field of the earth, whether or not the earth is rotating.[8] The
third term arises from rotation of the earth, and is analogous
to the rotation effect predicted in a different connection by
Lense and Thirring.[9]

     Measurement of the precession predicted by Eqs. (1) and
(2) would provide a new experimental test of general relativity
theory. At a conference on experimental tests held at Stanford
University in July 1961,[10] the late Professor H. P. Robertson
stated without proof the expression for the geodetic precession
(second term of Eq. (2)) in an arbitrary spherically symmetric
metric.[11] Following Robertson, we write the metric for the
nonrotating earth in the most general isotropic form:

$$ds^2 = [1 - 2\alpha(GM/c^2 r) + 2\beta(GM/c^2 r)^2 + \cdots]dt^2$$

$$- (1/c^2)[1 + 2\gamma(GM/c^2 r) + \cdots](dx^2 + dy^2 + dz^2), \quad (3)$$

which includes the leading terms of an expansion in powers of the
small parameter $GM/c^2 r$. The dimensionless numbers α, β, γ are
expected in general to be of order unity and are all equal to +1

in the Einstein theory. It should be noted that there is no loss
of generality in assuming the isotropic form, since if $dx^2 + dy^2 + dz^2$
is expressed in spherical coordinates, the radial and angular parts
can be given different coefficients by means of a transformation
of r.

Now as is fairly well known, the measurable quantities in
the three "crucial tests" referred to earlier are, in lowest order,
proportional to the following combinations of the α, β, γ that
appear in Eq. (3):[12]

gravitational red shift: α ;   (4a)

defelection of light: $\alpha + \gamma$;   (4b)

perihelion precession: $2\alpha(\alpha + \gamma) - \beta$.   (4c)

The number α not only determines the red shift in accordance
with Eq. (4a), but also is responsible for the leading term in
the gravitational acceleration produced by the mass M, and
hence for the orbits predicted by Newtonian theory. Thus α ,
or more precisely the product αG, must be regarded as very well
determined; with the conventional definition of G , α is equal
to $+1$ with great accuracy. The observational errors associated
with the measurement of the deflection of light are roughly 20%,[13]
so that γ is not very well determined from Eq. (4b). On the
other hand, the precession of the perihelion of the orbit of the

planet Mercury agrees with the prediction of general relativity theory within about 2%,[13] so that the combination $2\gamma - \beta$ is known from Eq. (4c) with this accuracy.

It is therefore of some interest to see how the geodetic precession of a gyroscope depends on α, β, and γ. To this end, the dynamical calculation of reference 4 will be generalized to the metric given in Eq. (3). We shall do this only for the geodetic term, and quote results only for the isotropic metric and the Pirani boundary condition. The pertinent formula is then Eq. (24) of reference 4, with the nongravitational acceleration f set equal to zero and the mass parameter m replaced by GM/c^2 :

$$d\underset{\sim}{S}/dt = (GM/c^2 r^3)[\underset{\sim}{S}(\underset{\sim}{r} \cdot \underset{\sim}{v}) + 2\underset{\sim}{v}(\underset{\sim}{r} \cdot \underset{\sim}{S}) - \underset{\sim}{r}(\underset{\sim}{v} \cdot \underset{\sim}{S})] \ . \tag{5}$$

A recalculation of Eq. (5) with the metric of Eq. (3) yields:

$$d\underset{\sim}{S}/dt = (GM/c^2 r^3)[(2\gamma - \alpha)\underset{\sim}{S}(\underset{\sim}{r} \cdot \underset{\sim}{v}) + (\gamma + \alpha)\underset{\sim}{v}(\underset{\sim}{r} \cdot \underset{\sim}{S}) - \gamma\underset{\sim}{r}(\underset{\sim}{v} \cdot \underset{\sim}{S})] \ . \tag{6}$$

Comparison of Eqs. (5) and (6) shows that they agree when α and γ are given the Einstein value +1.

It is necessary to express Eq. (6) in terms of the angular momentum $\underset{\sim}{S}_0$ measured by a co-moving observer. The relation between $\underset{\sim}{S}$ and $\underset{\sim}{S}_0$ involves a Lorentz transformation that is independent of the metric, and a coordinate transformation that

depends on the form of Eq. (3). Thus the first of these is the same as the Lorentz transformation given as Eq. (31) of reference 4:

$$\underset{\sim}{S}_0 = \underset{\sim}{S} - \frac{1}{2} [v^2 \underset{\sim}{S} - \underset{\sim}{v}(\underset{\sim}{v}\cdot\underset{\sim}{S})] \ . \tag{7}$$

On the other hand, the coordinate transformation involves the space part of the metric, and hence γ :

$$\underset{\sim}{S}_0 = [1 + 2\gamma(GM/c^2 r)]\underset{\sim}{S} \ , \tag{8}$$

which reduces to Eq. (25) of reference 4 when $\gamma = +1$. Combination of Eqs. (7) and (8) to first order gives the relation between $\underset{\sim}{S}$ and $\underset{\sim}{S}_0$:

$$\underset{\sim}{S}_0 = [1 + 2\gamma(GM/c^2 r) - \frac{1}{2} v^2]\underset{\sim}{S} + \frac{1}{2} \underset{\sim}{v}(\underset{\sim}{v}\cdot\underset{\sim}{S}) \ . \tag{9}$$

The time derivative of Eq. (9) is:

$$d\underset{\sim}{S}_0/dt = d\underset{\sim}{S}/dt - 2\gamma(GM/c^2 r^3)\underset{\sim}{S}(\underset{\sim}{r}\cdot\underset{\sim}{v}) - \underset{\sim}{S}(\underset{\sim}{v}\cdot\underset{\sim}{\dot{v}}) + \frac{1}{2} \underset{\sim}{\dot{v}}(\underset{\sim}{v}\cdot\underset{\sim}{S}) + \frac{1}{2} \underset{\sim}{v}(\underset{\sim}{\dot{v}}\cdot\underset{\sim}{S}), \tag{10}$$

where $\underset{\sim}{\dot{v}} = d\underset{\sim}{v}/dt$. It is sufficient for a first-order calculation to use the Newtonian approximation for $\underset{\sim}{\dot{v}}$. For the geodetic term, we again drop the nongravitational acceleration $\underset{\sim}{f}$, and note further

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that the gravitational acceleration must be multiplied by α
when the metric of Eq. (3) is used; thus Eq. (34) of reference 4
is replaced by:

$$\dot{\mathbf{v}} = - \alpha (GM/c^2 r^3) \mathbf{r} \ . \tag{11}$$

Substitution of Eqs. (6) and (11) into Eq. (10) then gives:

$$d\mathbf{S}_0/dt = (\alpha + 2\gamma)(GM/2c^2 r^3)[\mathbf{v}(\mathbf{r} \cdot \mathbf{S}) - \mathbf{r}(\mathbf{v} \cdot \mathbf{S})] \ . \tag{12}$$

As in reference 4, the difference between the differential
time intervals dt in the two coordinate systems may be neglected,
as can the difference between $\mathbf{S}$ and $\mathbf{S}_0$ on the right side of
Eq. (12). Equation (12) is thus equivalent to the geodetic term
of Eqs. (1) and (2), with the number 3 replaced by $\alpha + 2\gamma$. This
is in agreement with Robertson's conclusion that the geodetic
precession is proportional to $\alpha + 2\gamma$. Our derivation also shows
that the magnitude of $\mathbf{S}_0$ remains constant even when the general
metric of Eq. (3) is used. It follows that the gyroscope precession
experiment provides a method for the determination of γ that is
independent of the deflection of light; it is also slightly more
sensitive, since Eq. (4b) shows that the latter depends on $\alpha + \gamma$
rather than on $\alpha + 2\gamma$.

The magnitude of the precession angular velocity given in Eq. (2) is roughly 0.4" of arc per year if the gyroscope is at rest in an earth-bound laboratory, and carried through the earth's gravitational field by rotation of the earth. In this case, the three terms of Eq. (2) are of the same order of magnitude. If the gyroscope is in a satellite at moderate altitude, the geodetic precession is about 7" per year, and the precession caused by rotation of the earth is about 0.1" per year; in this case the gyroscope is in nearly free fall, so that the Thomas precession is practically zero. Since both of the experimental gyroscopes now under active consideration are intended for satellite use, it follows that the first result obtained will be an independent measurement of γ. Ultimately, it is hoped that the experiment will demonstrate for the first time, through the much smaller third term of Eq. (2), the effect of the rotation of a massive object on its gravitational field; this is also a prediction of the Einstein theory that has no Newtonian counterpart.

The more advanced of the two gyroscopes referred to above is the electric vacuum gyroscope.[14] It consists of an electrically conducting sphere that is constrained and supported by the electric fields between its surface and three mutually perpendicular pairs of close-fitting electrodes. As is well known, such support by electric fields is dynamically unstable, so feedback loops that adjust the field strengths in accordance with the sphere-electrode

spacings must be provided. This is accomplished by using alternating voltages in nearly resonant circuits with external inductances, so that the change in cpacity produced by motion of the sphere with respect to one pair of electrodes automatically changes the voltages in such a way as to restore it to the desired position. This approach can be extended to a three-phase electrical system, with one phase for each of the perpendicular electrode pairs; the sphere then becomes an electrically floating neutral. It seems desirable also for the gyroscope to have a slightly larger moment of inertia about one axis than the other two, so that it will spin naturally about this axis. Then the symmetry of the support is preserved if one of the three electrode pairs is maintained along the spin axis. Readout of the direction of the spin axis is being accomplished by an optical method that consists in viewing a sinusoidal curve etched around the equator of the sphere.

The second gyroscope consists of a superconducting sphere supported in a static magnetic field.[15] The sphere acts as a perfect diamagnetic, so that the support is dynamically stable and no feedback loops are required. Since low temperature is required in any event in order to maintain superconductivity, ambient electric and magnetic fields can be greatly reduced by using a superconducting shield. The low temperature also decreases thermal distortion since all coefficients of thermal expansion are then very small. The readout now being developed makes use of the Mössbauer effect. A small amount of a suitable radioactive material

is placed on the sphere, and the gamma rays from it pass through
an absorbing plate that rotates coaxially and synchronously with
the sphere. Any misalignment of the axes of sphere and plate will
result in a periodic change in the relative velocity of the two,
and hence a periodic change in the Mössbauer radiation measured
by the detector placed beyond the plate. Laboratory tests indi-
cate that this method of reading out the direction of the spin
axis of the sphere will have sufficient accuracy for the precession
experiment.

As remaked above, both experiments are planned for satellite
use. The principal reason for this is that the effective acceleration
of gravity in a satellite at moderate altitude (difference between
the earth's gravitational acceleration g and the acceleration of
the satellite) is extremely small; it arises from external forces
such as light pressure and atmospheric drag, and is probably of
the order of $10^{-7}g$. Thus the constraining forces required (electric
and magnetic in the two gyroscopes described above) are very small,
and extraneous torques that arise from these forces in conjunction
with imperfections in construction are hopefully small enough so
that they do not obscure the general relativistic precession. A
secondary reason for use of a satellite is that the precession to
be observed is much larger than in an earth-bound laboratory. On
the other hand, it is apparent that any experiment is more difficult
to accomplish and to monitor in a satellite than on earth; however,
this disadvantage is believed to be outweighed by the factors just
mentioned.

An additional refinement in the satellite experiment is also being given serious consideration. It was suggested by Pugh[16] and Sherwin[17] that the satellite be made to follow the gyroscope. This would require that the position of the gyroscope with respect to the satellite be sensed without exerting a force on the gyroscope, and that forces then be applied to the satellite so that it maintains a fixed position with respect to the gyroscope. There are three main consequences of such arrangement. First, the nongravitational force that must be exerted on the gyroscope is reduced from $10^{-7}g$ times its mass to zero, thus further reducing extraneous torques. Second, the satellite will follow a true gravitational orbit about the earth, and observations of it will provide precise information on the figure of the earth. And third, the forces that must be applied to the satellite in order that it follow the gyroscope may be interpreted in terms of atmospheric density. The last two would be useful by-products of the general relativity experiment that are of interest for geodesy and high-altitude meteorology.

Even without the "slaved" satellite described in the last paragraph, it seems likely that satellite gyroscope drift rates can be reduced to less than 0.1" of arc per year, and that the direction of the spin axis can be read out with an accuracy considerably better than 0.1". There would then remain the problem of relating the direction of the spin axis of the gyroscope

to some externally established direction, presumably that of a star. This would require that the satellite also contain a rather good telescope. According to present plans, it seems possible that a telescope of about one meter aperture will be in orbit by the end of 1965.

Footnotes

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[10] For a summary of this conference, see Physics Today, November 1961, p. 42.

[11] This result is also given without proof in a posthumous paper to be published shortly in the Journal of the Society for Industrial and Applied Mathematics.

[12] For a discussion of the physical basis of Eqs. (4), see L. I. Schiff, Jour. Soc. Ind. Appl. Math., to be published.

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